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ABSTRACT

A simple, rigorous, small-sample statistical technique is described for testing the hypothesis that two sets of measurements differ only because of errors of measurement and because of differing origins and units of measurement. (Author)

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TESTING IF TWO MEASURING PROCEDURES MEASURE THE SAME PSYCHOLOGICAL DIMENSION

Frederic M. Lord

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Frederic M. Lord, Principal Investigator

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TESTING IF TWO MEASURING PROCEDURES MEASURE THE SAME PSYCHOLOGICAL DIMENSION

Abstract

A simple, rigorous, small-sample statistical technique is described for testing the hypothesis that two sets of measurements differ only because of errors of measurement and because of differing origins and units of measurement.

TESTING IF TWO MEASURING PROCEDURES MEASURE

THE SAME PSYCHOLOGICAL DIMENSION*

This note is concerned with testing the hypothesis that two sets of measurements differ only because of a) errors of measurement, b) differing units of measurement, and c) differing arbitrary origins for measurement. It describes a simple, rigorous, small-sample statistical technique that, unnoticed, has become available (Villegas, 1964). Other techniques used for this purpose are cumbersome, approximate, or flawed (Forsyth & Feldt, 1969, 1970; McNemar, 1958; Lord, 1957).

Statistical Procedure

The necessary raw data consist of $r \geq 2$ replicate measurements (indexed by the subscript k) on each of N people (indexed by a) by each of two measuring instruments or procedures (indexed by g or h). For each instrument or procedure separately, compute the usual among-persons and within-persons sums of squares. Also the corresponding sums of cross products between instruments (or procedures). Let W denote the 2-by-2 matrix of within-persons sums of squares and cross products, and let A denote the corresponding 2-by-2 among-persons matrix. In a standard notation, the element of W in column g and row h is $\Sigma_{a=1}^{N}\Sigma_{k=1}^{r}(x_{gak} - x_{ga.})(x_{hak} - x_{ha.})$. A typical element of A, similarly, is $r\Sigma_{a=1}^{N}(x_{ga.} - x_{g..})(x_{ha.} - x_{h..})$.

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Let F_p denote the $1 - p$ percentile of the F distribution with N and $N(r - 1)$ degrees of freedom, for the numerator and denominator respectively. Compute the matrix

$$M = (r - 1)A - F_p W \quad .$$

From the product of the two diagonal terms of M subtract the product of the two off-diagonal terms, thus obtaining the value of the determinant $|M|$. If the determinant is positive and if both diagonal terms are also positive, then the F test rejects at significance level p the hypothesis H_0 stated at the beginning of this paper. Otherwise, the hypothesis is not rejected. This same conclusion may be stated more compactly as follows: H_0 is rejected if and only if the matrix M is positive definite.

Discussion

The assumptions required for the validity of the foregoing statistical procedure and conclusions are as follows. Within each replication, the errors of measurement for instruments g and h are random variables having a bivariate normal distribution. The mean error is always taken to be zero. No restrictive assumptions are made about the variance or correlation parameters of this distribution; in particular, the errors for g and h may be correlated within a replication. The within-replication bivariate distribution is the same for each replication and for each individual measured. Across replications, the errors of measurement are always uncorrelated with each other.

The unknown relationship between the two origins mentioned in H_0, also that between the two units of measurement, represent "nuisance parameters" for the purpose of making a significance test of H_0. Villegas shows that when (and only when) M is positive definite, there is no set of values for these (unknown) nuisance parameters such that an appropriate variance ratio calculated from them and from the data will lie below the p -level of significance.

Further developments along similar lines are reported by Walter Kristof (personal communication).

It is important in practice that the replications meet the assumptions stated. Any practice effect between replications that increases the within-persons sums of squares, for example, will tend to decrease the chance of rejecting H_0.

It is worth noting that Tukey (1951) accurately foretold the nature of Villegas' significance test. Tukey describes a very similar test and states

> "While not identical, the correction of correlation coefficients for attenuation is extremely close to our topic....(The writer believes that correlation coefficients have been highly 'efficient' in shielding the essentials of this and related situations from the inquiring eye. Unless and until we come to variances and covariances in measured, not standardized units, it is hard to meet the facts face to face.)"

Numerical Example

In the numerical example, measuring instruments g are unspeeded 15-item vocabulary tests; measuring instruments h are highly speeded

75-item vocabulary tests. The $r = 2$ parallel forms of each test were administered to each of $N = 649$ examinees. The raw data are the same as those used in a previous numerical example (Lord, 1957, pp. 210-212). The required sums of squares and products are shown in the accompanying table.

	Unspeeded Test		Speeded Test
	Sum of Squares	Sum of Products	Sum of Squares
Among Persons	93524.56	76176.30	111220.58
Within Persons	18533.00	-547.00	15403.50
Total	112057.56	75629.30	126624.08

The .05 significance level of F for 649 and 649 degrees of freedom is 1.13. The matrix M is easily found to be

$$M = \begin{bmatrix} 72582 & 76794 \\ 76794 & 93815 \end{bmatrix} .$$

The determinant is positive, so H_0 is rejected. This agrees with the conclusion reached previously by large-sample methods (Lord, 1957) under somewhat different assumptions.

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